Tension, Coping, Imagining

Introduction

In my previous post, I described a model of problem solving states based on the mobilisation and dissipating of free energy. This may be described by Gattegno as the work of the self.

I that post, I conjectured that it may be beneficial for the self to seek a return to a neutral 'contemplative' state through regulation of energy, thus enabling intuition - looking at the whole, maintaining complexity - leading to a greater possibility of insight.

In this post I will develop this model further with regards to tension, coping and imagining.

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In a newsletter *Problems and Solutions* [1], Gattegno defines a problem as follows:

A problem is experienced as an inner tension associated with other components of the mind related to other areas of living.

A problem brings with it a feeling that we must take steps to remove the tension.

What allows the tension to be removed is called a solution.

This definition of a problem in terms of tension to be removed fits with a model of mobilisation and dissipation of energy as follows: We become aware of tension (when solving a problem). This mobilises free energy in order to resolve the tension. Actions then become available that allow dissipation of energy, a movement towards a solution.

This is another way of describing an awareness as 'that which enables action'.

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In a 1983 seminar on *Awareness* [2], Gattegno started by asking the participants about 'coping':

Are we ever at the level of challenges that come to us? Do we ever have the sense that what's coming to us is going to be coped with? Even if we don't know what it is? Do we ever know what it is?

Exploring our own mechanisms for coping is useful in developing our own ability to solve problems (not necessarily mathematical). As teachers, we must understand our own mechanisms for coping in order to help students develop their own mechanisms when solving problems - particularly for those facing exam situations.

What might we need to understand about ourselves in order to help students develop their own mechanisms for coping?

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In this post I record some awarenesses concerning tension, coping and imagining that I discovered while working on three problems from John Mason's *26 years of problem posing* [3].

Problem 1: Tension

It is worth noting before starting that I chose this problem because it looked quite difficult - I wanted a challenge, to get stuck - but while choosing the problem I became aware of a tendency towards closed and immediately comprehensible ones, an implicit judgement of my ability to cope. More on this later.

Now to the problem:

Find Them All

Find all continuous positive functions on $[0, 1]$ such that $\int_0^1 f(x)\,dx = 1$, $\int_0^1 xf(x)\,dx = \alpha$ and $\int_0^1 x^2 f(x)\,dx = \alpha^2$.

You might like to try it before reading on, becoming aware of and recording felt tensions that mobilise energy (enable actions), and also then the subsequent steps you take to dissipate energy.

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As I worked on this problem I became aware of the following tensions, mobilisations and dissipations.

- Intially in a neutral stage, objectifying energy through examples, images, imagining possible approaches.
- Solve a simpler problem, a form of coping, remaining in a neutral state. Aware of my inner monitor present but allowing periods of ‘flow’.
- A sense of upcoming complexity, the first felt tension mobilising reticence, a need to employ intuition.
- Solution to the initial simpler problem not as simple as would have liked, creating tension as doubt, which mobilised checking.
- Checking revealed some further structure, a dissipation of energy / resolution of some tension, something else to work on.
- Increasing amounts of information to cope with. Tension mobilising a need for objectification: a desire for ‘clarity’ leading to organising my results systematically on the page, in the hope of finding further structure, coupled with an awareness of ‘too much happening’, mobilising a return to a more contemplative state, felt as a desire for ‘stillness’]
- Reticence to get involved in lengthy manipulations - a tension mobilising a search for structure, looking for an ‘easier way’, bringing other knowledge to bear on the problem.

Problem 2: Coping

Given the awareness that I choose problems that I feel I can immediately cope with, I decided to try to choose one that I felt much less able cope with on first inspection.

Here is the problem:

> Two buckets are connected by a rope over a pulley so that they just balance. A ball is dropped in one bucket and bounces several times. As it bounces, will the distance it travels be the same, shorter, or longer than the distance it would travel when bouncing in a stationary bucket?

Immediately there is tension present for me. It seems of the realm of mechanics, an area in which I am much less confident. There is a sense that I will probably not be able to solve this problem from the beginning.

This feels important for improving my understanding of the way my students might approach a problem they feel unable to cope with.

I start by trying to imagine what might happen. The bucket will start to go down as the balls strikes it, I think.

A flurry of questions come to mind. Will it have something to do with impulse? Perhaps something to do with elasticity (it's a long time since I worked on this), I can probably just set elasticity to be something simple. Perhaps it will be easier to solve using kinetic/potential energy, but I don't feel confident using this. Does it depend on how high I drop the ball? It must do, it's not mentioned, but I can again just assume some arbitrary height.

Much tension now, a felt desire to dissipate by doing… something! Let a = 10 for simplicity. I could assume a height, and use equations of motion to find the velocity of the ball as it hits the bucket, or even easier I can just let v = 4, say.

The impulse is then..? Let e = 1/2. Let m = 1kg… conservation of momentum is required to calculate the velocity of the bucket after collision… Let m for the bucket be…

The self is doing its best to react to what is coming, but with little faith that this approach will prove successful. If the bucket starts going down, the ball will have further to travel, but the impulse will be less when it hits it… and the velocity of the bucket after the collision will be affected by the other bucket on the pulley. I am sure I can calculate all this, but it is going to get complicated…!

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What is interesting for me here is that there is tension, and there is *tension*! We often only approach challenges we can cope with, or choose not to cope with others as they become too complex. Here I chose to cope with a challenge that I felt was beyond me from the beginning, which felt very different to what normally happens when I *choose* to do maths. I was *having* to deal with amounts and directions of energy that felt at times beyond my capacity to regulate.

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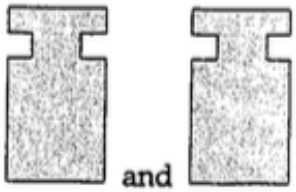
Problem 3: Imagining

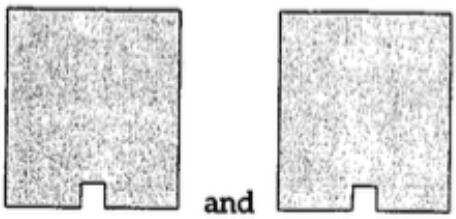
Now try this one. What questions do you have? Are you able to imagine what is happening? Are you able to start without imagining?

Shadows of an Object

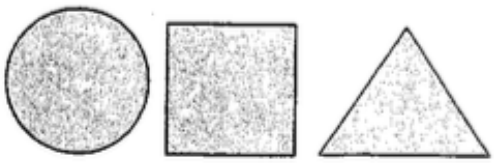
On a dark night after a Summer School Chez Angelique Mathematical Cabaret, I thought I saw the shadow of an object off to my right. I went a quarter of the way round it, and saw a second shadow. What was I looking at?



The same thing happened on a second night, but with a different object. What was I looking at?



On a third evening I got three views from three different directions:



Was I dreaming?

A piece of wire can be shaped so as to cast a shadow of a square in three mutually perpendicular directions. Can the wire be arranged to cast circular shadows in three mutually perpendicular directions?

If you are completely stuck, did you read all the way to the end of the problem? Does this ‘cast light’ on the problem?

Try imagining all of the objects, including the wire ones, without the help of any physical objects. Is this pleasurable? Try drawing what you imagine; is this pleasurable? Why do you think this might be?

What might this have to do with tension and the mobilisation and dissipation of energy?

Conclusion

Gattegno suggests that imagination is essential in order to cope with the future. Imagination is a mobilisation of energy from within, that makes virtual action possible. All of the following quotations are taken from *The Mind Teaches the Brain* [4]:

"Imagination is needed to recognize boundaries and their artificiality, and to remove them through invented exercises that bring to the center of awareness what needs to be done."

"The imagination says that we can make the potential actual… an act of the mind that transforms the given, producing what is compatible with it but serving a new function."

"*The imagination can supply a future for the present, and this alone makes it a special aspect of the self. This future, being possible, does not need to be actualized, or even to be probable, for the self to entertain it and find it valuable in its education."*

We could go further and explore the vital role of sleep in coping and imagination; I will leave that for another post. For the time being, I will ask:

What (mathematical) exercises can we give students to help them come into contact with the mobilisation and dissipation of energy, to educate their imagination, thus helping to develop within them the capacity to cope?