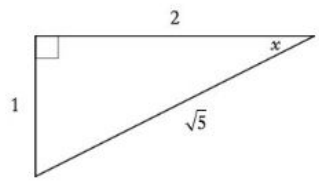
Diary: 26 September

Today J and K worked on [some problems involving trigonometric identities](). Here are five things I saw them doing whilst attempting to solve these three problems:

Question 1

The diagram shows a right-angled triangle with sides and angles as marked.



Find the value of $\sin 2x$.

Find also the value of $\cos(2x)$ and $\tan(2x)$.

Does $\sin^2(2x) + \cos^2(2x) = 1$ in this question?

Is $\sin^2(2x) + \cos^2(2x)$ always equal to 1, for any x?

Question 2

(a) Find an equivalent expression for $\sin(x + 60)^\circ$.

(b) Hence, or otherwise, determine the exact value of $\sin 105^\circ$.

Show that $\sin(105) \times \cos(105) = -\frac{1}{4}$

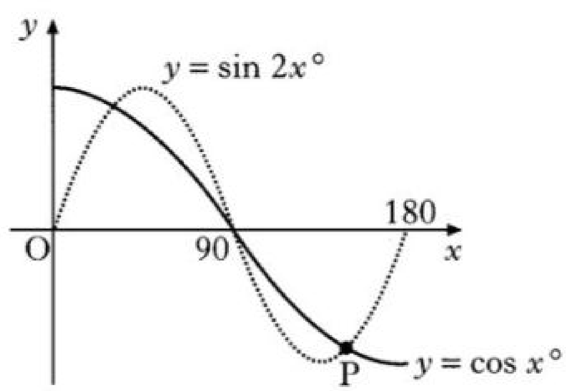
Deduce that $\sin(210) = -\frac{1}{2}$

Question 3

(*a*) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$.

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (*a*) to write down the coordinates of the point P.



It should be noted here that they had not solved any problems 'like these' before.

#1 Using a method that has worked in the distant, or very near, past

For the first part of question 1, both students initially tried to find an expression for x using inverse sine, presumably due to an association with problems about finding the missing angles in right-angled triangles.

One of them then labelled the side of length 2 with $\sqrt{5}.cos(x)$ and the side of length 1 with $\sqrt{5}.sin(x)$. This idea, of expressing side lengths in terms of $sin(x)$ and $cos(x)$, was a focus of last lesson.

Students often try to use what they have *just* learned to solve the 'next' problem, regardless of considering whether it is appropriate. A kind of 're-setting' seems to be required after learning something new.

After they had solved the first problem, they proceeded through the other parts of question 1 relatively easily.

#2 Looking for (and using) a method that 'always works'

Connected with #1 is an (over-)reliance on a single method, one that has proved reliable for a range of problems. I described in yesterday's diary entry how one student used the sine rule for many problems involving triangles, including those with a right angle.

Looking for a single method that works all the time is economical in one sense, but may not lead to the most efficient solution, or at worst, no solution at all. Another issue is that an over-reliance on one method may prevent the development of other methods.

#3 Being creative in the face of uncertainty

There were a number of instances of students finding novel solutions to problems.

For the second question, one of the students answered the question: *'Deduce that* $sin(210) = -\frac{1}{2}$*'* by expanding the addition formula $sin(180+30)$. I had not envisaged this elegant solution.

We had not yet solved any equations like that presented in question 3. One of the students gave the answer 90° almost immediately. Surprised, I asked how he had got this, thinking he must have substituted $x = 90$ into the equation. He had looked at the graph. He then asked whether he could work out the other solutions *"just by looking at the graph"*.

There were a number of examples of students *creating rules and methods*, such as writing $sin(210) = sin(105) + sin(105)$, and $tan(2x) = \frac{sin(x)}{cos(x)} \times \frac{sin(x)}{cos(x)}$.

Another example was to solve the equation $2.sin(x).cos(x) - cos(x) = 0$ by dividing by $cos(x)$.

Although these are examples of false (or partially correct) reasoning, it is just this type of creativity that is required in solving unfamiliar problems. What is then required is to develop the habit of checking whether a created rule or method makes sense, a way of ascertaining whether it is mathematically sound.

#4 Wanting something to be true, so thinking it is true

Whilst solving the question *'Show that* $sin(105) \times cos(105) = -\frac{1}{4}$*'*, a student wrote $\frac{\sqrt{6}+\sqrt{2}}{4} \times \frac{\sqrt{6}-\sqrt{2}}{4}$ followed by $\frac{\sqrt{12}-\sqrt{12}}{4} = -\frac{1}{4}$.

This is perhaps a case of wanting something to be true, and so thinking it was true. Or it might have been a case of fudging the answer.

#5 Looking to another for help / assurance

A number of times I was watching students working, and noticed that they would often write something and then look at me, perhaps for assurance that what they had written was correct.

Others times, they asked: *"Am I allowed to…?"* An example of this was whether they were 'allowed' to write $tan(2x) \equiv \frac{sin(2x)}{cos(2x)}$.

I am aware that my presence, my watching closely what they are doing, affects how they engage with problems.

Summary

I have described a number of behaviours exhibited by these students when faced with unfamiliar problems.

It is my experience that these are things that most students (and perhaps all humans?) do when faced with uncertainty.

What are the implications of each of these behaviours for teaching?