

BEING ALONGSIDE

Laurinda Brown and Alf Coles have a conversation arising out of reflections on a classroom incident.

Laurinda: I was sitting at the back of your classroom recently and you were using what we refer to as the Gattegno chart:

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

This Gattegno Chart can be downloaded at www.atm.org.uk/mt/

Your Y7 group was chanting, "... three point two; three point four; three point six; three point eight; three point ten".

Your pointer was following their chant by a fraction of a second and you looked around with nowhere to point. I remember laughing – enjoying the moment; "... four; four point two; ... it was five next time around.

I stood in front of my new group of PGCE students and, given this experience, it felt like some chanting would be good: "... three point two; three point four; three point six; three point eight; four; four point two".

No laugh this time. I felt sad. Rather than them experience chanting exposing a problem through saying "three point ten", I'd have to tell them a story from the classroom instead. It was only on reflection afterwards that I realised that I could have raised the level of complexity until there was something to work on at their level. I had an attachment to "3.10" happening, wanting it to happen. Where did the energy come from when I was at the back of your classroom?

Alf: One story I am sure we have got to before, about the kind of energy you describe in my classroom is that it occurs when you expect sameness and get difference, or when you expect difference and get sameness. I think this occurred on many different levels in the incident you describe. For the students, the pattern (sameness) clearly broke down, "2; 4; 6; 8" and then, as you said, there was

nowhere to point for "three point ten" – this was different. I imagine some students who in another way 'knew' that 3.8 add 0.2 was 4 had found themselves saying "three point ten" because they had been taken along by the rhythm and the chanting. Was your laughter a recognition that this has also happened to you in some way? Then there is your expecting the same reaction from your PGCE students and getting a difference. You reflected on this afterwards.

Laurinda: Mmmm... I don't know. But I do remember writing down a time later in the lesson when you stressed the saying of "three point one zero" rather than "three point ten" because of its sounding bigger than "three point two". This seemed to arise naturally. I felt comfortable at that point. I do also recognise the power of the chanting in taking me along – power that can be useful in tripping students into doing something they didn't think they could do... I giggle, releasing energy, similarly with the chanting of "1; 2; ten; eleven; twelve; twenty; twenty-one; twenty-two; ...?" when, at this point, the group chanting pauses, seemingly swallows (gulps), and then "a hundred" comes out!

Have you got any other stories?

Alf: I've just realised you are talking about chanting in base 3 here. I have never done this but you have described before how the Gattegno chart can be used powerfully for that by ignoring, in this case, all but the first two columns. I have images of observing you working with students throughout the years of working in lessons together and being struck by how you somehow 'get alongside' the students. It is as though the 'mathematical' part of yourself is left in abeyance as you engage with the students' world (of mathematics). I remember one time working with you and some low-attaining Y9 students on halving. You were working with one or two individuals and, again, I think I was getting the whole class chanting, "halve 20; 80; 60; ...", all done smoothly and then, "70". No response, seemingly impossible! You described later also finding yourself in the position that halving 70

seemed a very difficult task. I now recognise that sense of empathy (for want of a better word), but I remember at the time being amazed that you didn't still just 'know' the answer was 35. Of course you did know, the issue is where, as teachers, we place our attention. And, particularly with very low-attaining students, any sense they get of their thinking being judged or evaluated is destructive of their capacity to learn. I think there is something

here about why generally you leave students energized about their work after 1-1 interactions.

Laurinda: I remember that time because of what happened afterwards. I've found some writing we did about that lesson, which formed part of an article for the *International Journal of Educational Policy, Research and Practice* (2001, Volume 2, Number 2). I've edited it a little.

A lesson with a Y9 special needs group

The lesson began by beginning to discuss the homework, which had been plotting a graph for the rule $n \rightarrow 3n + 1$. Another function game was begun:

2 \rightarrow 4
5 \rightarrow 10
10 \rightarrow

The pen was passed and if the answer was correct then the student who wrote it was invited to give a new starting number.

There was a discussion of what to do with $290 \rightarrow$. Bert said, "Might as well have been 300". Others did $290 + 290$ set out underneath each other and a few were doubling from the left. The fact that ' \rightarrow ' represented 'double' seemed not to be a problem and yet there was energy to tackle the numbers that others had written. They do not see finding the rule as the only thing they are to do apparently. Quite rapidly quite big numbers were being given as challenges. There was a sense of them playing with numbers as if they felt powerful. The ones who were working from the left seemed to get the answers very quickly. What were they doing?

1475 \rightarrow ~~2894~~50 which is 2950
21458 \rightarrow ~~42890~~16 which is 42916
6,532,198,456 \rightarrow ~~123,064,23896,890~~12
which is 13,064,396,912

From left to right, doubling meant that sometimes there had to be an adding of one to the number on the left when carrying was necessary. Carl and Stan, who were using this method, simply crossed out the number they had last written down, added one to it and carried on, rewriting the number afterwards without the crossed out numbers.

They enjoyed trying to say the numbers. Some were still amazed that they were working with these large numbers or thought that they should not be able to do it.

Bert wrote a number on the right-hand side –

a big one: 1748209 and put an arrow pointing to the left hand side \rightarrow . He was laughing with enjoyment. That's halving. How to do this? Carl said it was wrong – needed the commas – and they worked at how to put them in.

This had come from the group – big numbers were part of them since they had worked on a project to do with this idea earlier in the year, but here they were using and applying those ideas for themselves. And now, something else – inverse – an awareness of some level of structure? This can be done backwards? Or could it?

From left to right, doubling meant that sometimes there had to be an 'adding of one' to the number on the left as the carrying was necessary. Left to right halving suddenly made sense to Laurinda in a new way, as the opposite of what she had just been trying to work with. Now when there was something left over it meant adding tens to the next number to the right rather than adding one on to the left as we had done before. These two processes felt like they fitted into an inverse pattern and focused attention on carrying – the standard algorithms seemed redundant.

Not all the pupils worked in this way but there was a sense of learning from these children who are considered weak at mathematics and the whole group worked for the complete lesson on halving and doubling of very big numbers. At one stage, one of them asked for a calculator and there was another laugh of pure enjoyment as they realised that the numbers were too big and they had a good chance of doing it without anyway.

This had not been what had been planned. They were in the middle of some work on graphs and the function game was there to generate a rule to plot. Going with their interpretations had led to lots of practice at arithmetic and they had been demonstrating an awareness at some level of the connections between doubling and halving and another way of looking at inverses that had allowed the possibility of seeing the way the structure of carrying operated in the two directions. There were new awarenesses of algorithms for the teachers too.

Laurinda: There was a lot of energy for me in this – I like your use of the word ‘alongside’ because, as Carl was working, I was striving to know what he knew, and was, literally, sitting beside him. Here was a low-achieving student who was developing an algorithm that worked for him, working from the left, which would achieve something he had not been able to do before. It is the energy changes I remember; his focused attention and absorption. My attention, lending him that as I tried to follow what he was doing and his joy at being able to do and tell others the method. It is so much more natural to start working from the left.

Alf: Reading this lesson write-up again, there seem to be many moments of energy change that don’t easily fit with the model of sameness-difference or difference-sameness. The story makes me think that maybe any learning carries energy, as I become aware of something I was not aware of before or can do something I couldn’t do before. It seems significant that the students drove some of the key moments in the lesson above; eg the shift to big numbers and the inverse function. When I began teaching and became interested in the work of ATM and its founders I don’t think I had any images of what it meant for a class to be ‘working mathematically’. I have phrases like ‘authentic’, ‘deep’, ‘real’ in my mind to describe the learning these students were experiencing, but I don’t really know what these words signify except a feeling in my stomach that these students were in a place where they knew they needed this. It reads as though the students were enjoying how powerful they had become in this context, surprising themselves.

I remember often finding that group difficult to teach. You and Dick Tahta observed them once around that time and I remember Dick talking about “trampling on flower beds” in describing some of the students’ behaviour. This phrase has been in my mind recently, working with some teachers new to the profession. It nicely captures for me something about the value of boundaries and why they can be important, or at least what can be destroyed if they are not respected. I entered teaching coming from a naïve position of not wanting to impose boundaries. It’s not in the write-up but there was energy for me in seeing how mathematically these students could work, and perhaps some confirmation of Gattegno’s discovery that everyone can access the extraordinary learning power we all used as infants.

I am taken by your phrase, above, “lending him [my attention] as I tried to follow”. Some questions spring to mind: Is this a conscious

decision you make? Is it a description of how you always work with others? Is it necessary to follow in order to lend?

Laurinda: Interesting play on words as I think about follow. I had had in my mind that I was making connections in relation to the development of the mathematics of the boy, and reading your comment thought of ‘follow my leader’. Attention is important in this and I also know that I interject if I don’t ‘follow’, which, as you so perceptively say, is not to do with whether I could go off and do this twice as fast in my own way. Not following is if I don’t make a connection from where we are and what has been said by the other. And this is back to listening whilst being vulnerable to being changed and I get the sense that I am moving away from the experience. Dick Tahta talks about some things being unsayable and this feels like one of those. If I try to say more in relation to your questions I seem simply to get further away from the experience. This conversation has been useful for me, though, because being alongside is something about paying attention to the energy changes in me as I attend. I can’t be responsible for those in the other. I am striving to know so, if I’m blocked I’ll ask for clarification and if I make a connection I check it out to see if that is what the other saw.

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It was with great sadness that we received the news of the death of Dick Tahta in early December.

A former editor of MT, he was a man of great compassion and integrity and will be greatly missed by all who knew him.

A full obituary will appear in the March issue of this Journal.

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