

An approach to solving problems

Can we teach problem solving? Not distinct from solving problems, certainly, but we may be able to reach what Gattegno calls the 'dynamics rather than the content' when solving a problem.

What follows is a description of an approach which may lead towards this.

The approach

Starting with a blank sheet of paper, draw a vertical line dividing the page in two. On the left you will 'do the maths' and on the right you will record awarenesses *beyond* the content.

What I am calling awarenesses here are (possible) actions, thoughts, emotions that 'come to the level of awareness' whilst solving the problem, recorded *as you become aware of them*. They may only be available for a split second; it is crucial not to let the content 'take over' in these moments.

In order to remain with the dynamics and not the content, I have found it useful to keep at the front of my mind what this exercise is primarily for: the stressing of awarenesses rather than the solving of the specific problem. That is not to say one does not attempt to solve the problem, but rather that one does not get 'lost' in the solving of the problem.

It is preferable that one gets stuck on a problem for it to provide a richness of experience. It is necessary that one is honest with oneself. Getting stuck is a valuable opportunity to find out something about oneself, and is not to be missed.

That said, it is of some note that one's ability to record awarenesses diminishes as we find ourselves lost in the content; should this happen, one may wish to mark that this has occurred.

A problem

I outline below my awarenesses whilst solving this problem taken from John Mason's '26 years of problem posing'. I would invite you to try it for yourself first, and record your awarenesses.

Random Integral Division

If $n + 1$ distinct numbers are chosen at random from $\{1, 2, \dots, 2n\}$, show that one of them divides another integrally.

If you find this one does not provide the sufficient level of challenge, you might like to try the approach with a different problem that results in you getting 'stuck' for a reasonable period of time.

My awarenesses

Here is what I recorded on the right hand side of my page as I did this problem (in time):

- specialise - simplify as far as is possible/useful
- possible structure
- vague conjecture
- contemplation required at first fuzziness - recognition of possible complexity
- looking for structure
- shelve idea for later
- testing conjectures
- image (energy objectified?)

- conjecture
- refinement (of conjecture)
- looking at cases
- sense of deeper complexity
- looking at specifics whilst keeping a sense of whole/general in mind (how does his example help me *know*?)
- vague conjectures
- contemplation
- change of image
- seems to be true, but why? intrigue/mobilisation of energy
- contemplation (enjoyment/intrigue)
- vague conjectures
- is it possible to count this?
- argh! this approach not sufficient
- increasing complexity leading to action: needing to contemplate whole again... come back to original ideas? re-read question as 'fresh start'
- conjecture
- in danger of getting lost
- looking at cases
- vague conjecture
- vague conjectures
- specialise again, build up pattern, look for structure
- mmm...!
- desire to leave for a while
- change of approach required... or go back? did I fully explore all of my ideas?
- complexity/difficulties (with original approach)
- opening/'mulling'
- following something suspecting it is probably not useful in hope of some pattern
- no pattern here, lack of idea, possible emotional response here... contemplate again
- re-read question / fresh start
- going back over old ground
- vague conjecture
- consider whole - contemplate
- conjecture
- sense of current approach not going to be successful, but in absence of any other...
- losing appetite, time constraints (other things to do), questioning whether this is an economical use of my time and energy, what am I going to learn by carrying on, what is the value to me?
- trying various images / layouts
- (one of the images suggests) a structure! conjecture...
- knowing / pleasure
- justify
- creating language to describe (something mathematical I can't recall seeing before)
- do I know? yes? can I convince another? sort of, I think so... yes. deeper knowing.

Conclusion

How do your awarenesses compare with these? What common themes / ways of knowing are present here? What might we / students learn from doing this?

It might be beneficial to begin to gather themes after trying this exercise a few times, perhaps with others. Some of these awarenesses have come from reading Gattegno's 'Science of Education' and Mason's 'Thinking Mathematically', alongside some of my own thoughts and feelings.

This approach might prove useful in developing an inner monitor, and thus in solving mathematical problems; it may also have further applications beyond solving maths problems.