METAPHOR AND METONYMY

Alf Coles reflects on how Dick Tahta and Caleb Gattegno demonstrated that mathematics teaching could be powerful, playful and relevant.

The map is not the territory (Bateson, 2000). A name is not the thing named. My use of language necessitates a transformation of my experience.

Visualise a cube. You may know the number of vertices, edges and faces, but try to see them – count them off from your image.

In a classroom recently, I worked with my students on the challenge above, agreeing the number of faces, edges and vertices in a cube. Students attempted to articulate the image they conjured. As a group, we developed some words that seemed to be meaningful and allowed us to agree on certain features common to the images each of us had invoked. Somehow we were able to move between words and image in a consistent manner.

As I re-read the paragraph above after a gap of a few days, I am aware of an idea I had in mind while writing, that I was trying to articulate, and that reading now does not quite capture. We are continually faced with circling transformations between ideas, images, language, text, such as in the example above, as we attempt to bring meaning to our lives.

> 'Now if we consider the matter more diligently, perhaps you will find that there is nothing that is learned by signs proper to it. For when a sign is presented to me, if it finds me ignorant of the reality of which it is a sign, it cannot teach me anything; but if it finds me knowing the reality, what do I learn by means of the sign? … we learn the meaning of the word – that is, the signification that is hidden in the sound – only after the reality itself which is signified has been recognised, rather than perceive that reality by means of such signification.' (St Augustine, 1978 edition, pp.173-4)

This is a complex passage but worth struggling with. I take sign here to mean any symbol (such as a word) that stands for, or indicates, something else to someone. The 'something else' is what I take St Augustine to mean by 'reality'.

In working with a class of students on their visual images of cubes, the new word 'vertex' becomes meaningful as individuals recognise aspects of their image that others seem to be indicating by their use of that word. The word or sign 'vertex' becomes a metaphor for this visual abstraction. The word comes to stand for the abstraction and, after time, can be used, if necessary, without invoking any image. A student in the class then questions the use of 'vertex', saying she doesn't know what people mean. Another student offers that it is the same as a 'corner'. In this instance 'corner' is used as a metonym for 'vertex', a substitution of one sign for another.

Metaphor and metonymy are two ways we make meaning; two ways we perform transformations between thought and language and within both thought and language. In St Augustine's language these two processes could be pictured as vertical or horizontal movements either from the world of signs to 'reality' or within the world of signs itself (*Figure 1*).

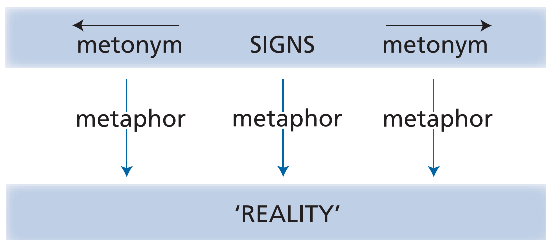

Figure 1

In my own mathematics teaching, one of the messages I heard from ATM publications, as I began work in schools over 10 years ago, was the importance of students experiencing why mathematics works in the way it does. I was keen on metaphors – trying to give students images or objects on which to hang new words. I remember a powerful experience on my PGCE course of being in a session where Charlie Gilderdale invited us to work with *Multilink*. I had an awareness of how powerful this could be in allowing students to connect with what volume actually meant. This session was part of my re-awakening interest in mathematics; I began to reconnect and re-engage with the subject after the (seemingly) purely metonymic abstraction of university mathematics had left me silenced.

Negative numbers

A common experience for secondary PGCE students on teaching practice is to face the difficulties of teaching arithmetic with negative numbers: why is $-1 \times -1 = 1$? (my ex-head of maths' favourite interview question!). I remember, faced with this, offering students images of putting hot and cold bricks into cauldrons. Taking out a cold brick (subtracting a negative) would make the cauldron hotter ($--1 = +1$; voila!). I was searching for metaphors, hopefully close enough to students' experience, with which they could connect the behaviour of these signs and symbols.

PGCE students on teaching practice at my school will usually ask me at some point how I teach negative numbers. My, now standard, reply (as recorded in an *MT* article – Durdy, 2002 – by a PGCE student at my school) is that I don't teach negative numbers. What I do is work with students on complex activities in which they are forced to use negative numbers, in a context where they get feedback that allows them, perhaps, to create their own metaphors for what is happening. One such activity is working with functions and graphs.

Students work on the challenge of predicting what the graph of any rule will look like, without having to plot a series of points (see www.mathsfilms.co.uk for a more detailed write-up of this activity and others). Typically, in Y7 most of their work focuses on linear and quadratic rules (eg, $y = 3x + 2$ or $y = x^2$). Students choose their own rule or sets of rules, work out a series of points for that rule by substitution (always including some positive and some negative numbers), plot the graph(s) and then look for patterns. They pin up completed graphs on a wall so that everyone has access to the search for patterns and connections. Typically, students become convinced that linear functions should be straight lines. This introduces an element of self-checking: as students substitute negative numbers into their rule, they can see if the point they get is consistent with the others. The structure of the situation, of the mathematics itself, gives students a visible metaphoric context within which they can check out their emerging understanding of how negative numbers operate. Within this context, I offer only metonymic support to any student who asks me about negatives. For example, if a student asks: "What is -1×5?", I might reply: "To me, that sum is the same as working out 5 lots of -1". Or else I might ask them to try out on their graph what they think the answer should be and see what happens.

I now think there is something unhelpful about images of hot and cold bricks. Learning the rules of negatives becomes the end point of the activity, and the rules can seem arbitrary because the metaphor itself was arbitrary. Working on functions and graphs provides a context for working *with* the signs (metonymically) rather than worrying about what they 'mean', and what I find is that students develop meanings over time that are robust.

Place value

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000

Figure 2

I like to make sure at the start of each school year that there is still a Gattegno chart (*Figure 2*) up in every classroom in the maths department. Last year, we had a discussion with one teacher saying they thought they would use the chart a lot more if it was rotated 90 degrees clockwise, so that the columns represented the place value position of the numbers. This suggestion got me thinking why Gattegno had arranged the chart in the way he had. My suspicion (which Dick Tahta agreed with, in conversation) is that the arrangement above allows the numbers in different rows to be literally placed over each other. I can tap on 1000, 100, 10 and if I were able to merge those three numbers on top of each other, I would see 1110. The emphasis, in this arrangement, is on the metonymic playing with number names, rather than the metaphoric 'meaning' of place value.

Gattegno did not feel it was necessary to teach place value, as such. For example, try to watch yourself work out: $200 + 300$. Gattegno felt that the 'hundred' part (or 'thousand' or 'million', etc) is often seen by students as some kind of trap. Rather than teaching students to be able to do sums such as these by knowing what a hundred 'means', Gattegno advocated a more linguistic approach:

> 'You can go on thinking of number – but it is also a noise. It's a number in a certain context. But when I add two hundred and three hundred, it is not a number that I add; it's two and three – hundred. By the distributive law, it is 2+3 hundred.' (Gattegno, 1991, p.99)

Indeed any arithmetical sum can be read as a (metonymic) transformation. 237 × 16 is a perfectly good way of referring to a particular number; as is 474 × 8 and 948 × 4 and 3792 × 1. I have often wondered why, in videos I have seen, Gattegno uses the equivalence sign '~' rather than '=' when working with number; it now seems to me that this was to emphasise that number work can be read as a series of linguistic (or metonymic) transformations.

Canonical images of mathematics

I would like to invite you now to spend a few moments trying to recall how you were taught to solve quadratic equations. My conjecture is that you cannot; and that this would be true of most introductions you had to mathematical topics. So, why do I sometimes feel the need, particularly at A-level, to labour the introduction of new ideas with careful proofs of where techniques have come from, when all students retain is the technique itself?

This is not a plea for a return to images of mathematics teaching where rules are pulled out of the teacher/magician's hat and the students' job is to practise them – but a statement of where I am at currently in re-thinking much of my practice as a teacher. It is as though, after 12 years' teaching, I am getting to, what seem to me, the key questions. I only now realise that Dick Tahta, in odd conversations over the last decade, was consistently trying to get me to re-consider my focus on understanding/meaning/reality (metaphor) as against an approach based more on language/play/transformation (metonymy). This distinction seems to me now a vital part of Gattegno's legacy.

Unlike my hot and cold bricks, some images are not arbitrary. One example is the image of a dot going around a unit circle, as a way of introducing trigonometry (*Figure 4*: see www.mathsfilms.co.uk for an animation of this). The sine and cosine functions can be introduced as names for the lengths of the blue lines – these functions simply tell you the vertical and horizontal distances from the moving point to the axes, relative to how far it

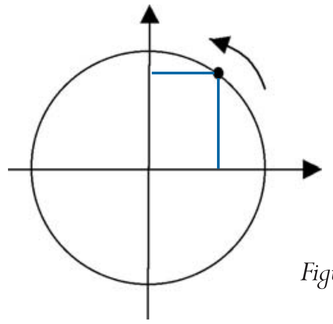

Figure 4

has moved around the circle. The sine function really is just the changing length of that vertical line (assuming we are dealing with a unit circle). Dick called this image, and the Gattegno chart above, a canonical image of mathematics. One of Dick's oft-repeated challenges was for someone to find the minimal set of such images that students need to be exposed to in order to be able to generate all the school mathematics they need.

Whereas in the past, when working with students on this image, I have moved fairly quickly into trying to use the image to solve fairly standard trigonometric problems, it now seems to me that a more powerful approach would be to spend time working with transformations of the sine and cosine of an angle, as indeed I recently saw Dave Hewitt do in a session (see Hewitt, 2007, 6). For example, students could work not just on what angle makes the sine line ½ (Dick's essential question for this image), but on what are all the angles are for which this is true (ie, 30°, 150°, 390°, etc). Students could usefully develop alternatives for sin $(-x)$, sin $(x + 180°)$, sin $(x + 90°)$, cos $(x - 90°)$ and so on; and then use them to transform expressions such as cos (490°) – perhaps they are only given the values of sin x with x between 0° and 90° in a table, with no access to calculators. I imagine this kind of work could be done in a playful manner.

Final remarks

Dick wrote on metaphor and metonymy far more lucidly than I have done here (see Tahta, 1998, available online at www.atm.org.uk/mt208):

> '[M]athematics itself seems to need to get away from metaphor as soon as it can' (p6);
> 'the move towards metonymy is sometimes seen as the terrifying abstract price that mathematics demands' (p7);
> 'Yet, mental and emotional reality may be otherwise, and there is accumulating evidence that babies are involved in abstract processes from the very start' (p7).

These, then, are the powers of children that Gattegno wrote about and so powerfully exploited in his teaching. I was recently lucky enough to re-see a video recording of Gattegno working on fractions with a group of 6-year-olds, using *Cuisenaire* rods (still available from Education Solutions, Reading). Gattegno begins working with the students to check they are confident recalling the lengths and colours of the 10 Cuisenaire rods (establishing the metaphoric context). He moves them on to finding which rods will fit exactly three times along the length of the 'nine' rod, and then two lots of the 'nine' rod, and then three lots. Students are offered something visible and tangible to which they are always able to refer if they get stuck in the later activity – and Gattegno indeed consistently refers them back to the rods whenever anyone asks a question.

Gattegno shifts the students into using their new-found powers of finding a fraction of multiples of 9, as they discover what else they are now able to do. At one point, he writes up four questions for the students to try and work out. When I was watching the film, at that point in the lesson, if I judged it was time for some questions, I know mine would have been along the lines of: Find ⅓ of 12; Find ⅓ of 39; Find ¼ of 8; Find ⅕ of 20, etc. Gattegno wrote up the following set of questions:

$$\frac{1}{2} \times (36 - 18) =$$

$$\frac{1}{3} \times (18 + 9) =$$

$$\frac{1}{4} \times (9 + 27) =$$

$$\frac{1}{2} \times (9 + \frac{1}{3} \times 27) - \frac{1}{4} \times 36 =$$

There is extraordinary complexity here, which, I imagine, would be very energising as a learner if I could cope with it, and the evidence showed that these 6-year-old students coped fine. And at the same time, he has stayed exclusively with finding fractions of the numbers 9, 18, 27 and 36 – the numbers with which students had developed their awarenesses of fractions. Staying with the one 'metaphor' of placing identical rods against the 'nine' rod, Gattegno invites a metonymic playfulness as students discover what they can now do, while also moving to abstraction.

In ***MT203***, John Hibbs quoted his favourite metaphor (!) for the teaching of mathematics:

> *A teacher took a class of little children into a wood, caught a toad and put him into the middle of the group, saying: "Our lesson today is to be about toads. But (as) I know nothing about toads; someone else will give you the lesson."*
> *One child said: "I suppose God will give us the lesson."*
> *Another interposed: "I think the toad will."*
> (Mary Boole in Brookes, Love et al, 1980)
> Let the mathematics do the talking, I would add.
> (Hibbs, 2007, p.30)

In the classroom examples I have quoted, there is something visible and/or tangible for students to return to (like Hibbs' toad) whenever they have need, and from which students are invited to draw distinctions. New ideas are worked on in as many different ways around as possible, inviting a linguistic, metonymic playfulness, as opposed to a laboured metaphoric derivation. The challenge, it seems to me, is to make the *toad* the mathematics itself, not some spurious or arbitrary context designed to be 'meaningful' for students.

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The 11+ disaster

In primary school the hardest part would be the 11+ in Primary 7. The work starts in P6 sometimes even Primary 5. The pressure is immense, sometimes if your classmates get all A's then you can feel down. So I am a P7 student who just got her results and I was lucky enough to receive an A grade. Some of my friends received help from a tutor but that is none of my business. I think that the 11+ is a very bad way to choose the school you are going to when you reach the time to transfer schools. The test is cruel in some ways, but in other ways it can also be fun. The practice tests were a lot easier than the real test. The first test mostly consisted of mathematics. The other two subjects were Science and English. The second test was, I think, the easier of the two. The 11+ can sometimes be a bit hard for the younger students. When I took the test I was only ten so I found it harder to store all of the information I had to know. Some of my classmates did not receive an A grade but they are going to their chosen school. The whole thing was a rotten way to tell students what school they are going to. I think that the best way of doing it is going to the school closest to you, it is a much easier and less stressful way of telling the pupils where they will spend the next seven years of their lives. The whole process exists for two to three years with the added month or two you have to wait for the school you have chosen to get back to you.

So the 11+ is not a great idea, it makes children grow apart after seven years of being together and makes some of them feel second class, which is not a good thing at our age.

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