Bi-cimals

In binary-decimals (bi-cimals), the first bi-cimal place represents ½, the second ¼, and so on…

Part 1

Show that $\frac{1}{7} = 0.001001001...$ in bi-cimals.

Find 2/7, 3/7, … up to 6/7 in bi-cimals.

Did you use any shortcuts to work these out?

What do you notice about your answers?

Fun aside

The fact that $\frac{1}{7} = 0.001001001...$ implies that:

$$\frac{1}{7} = \frac{1}{8} + \frac{1}{8^2} + \frac{1}{8^3} + ...$$

Can you make/prove a conjecture about infinite series of fractions 1/n?

Two facts

Convince yourself that any real number between 0 and 1 can be written as an infinitely long bi-cimal.

Convince yourself that all rational numbers can be written as periodic bi-cimals.

Part 2

Let x be any (infinite) bi-cimal. Can you describe what happens to digits of x following the following 'transformations':

(a) $2x$

(b) $1 - x$

(c) $1 - 2x$

(d) $|1-2x|$

(e) $1 - |1-2x|$

A puzzle

An infinite sequence of numbers x_n is determined by the formula

$$x_{n+1} = 1 - |1 - 2x_n|, \quad 0 \leq x_1 \leq 1.$$

Using bi-cimals, or otherwise, find out for which numbers this sequence is periodic, and which it is not. [Hint on next page if needed]

Hint for the puzzle

Assume that $x_{n+k} = x_n$

Can you use the results from Part 3 to show that the *digits* in the bi-cimal representation of x_n are periodic? What can you then conclude?