Analysing some of the harder problems

It is important that we get as much as we can out of solving any problem. It is not that useful just working through a set of exam questions without learning as much as we can from them, particularly the ones that we find difficult.

When we encounter a problem that we find difficult, there are things to learn about: (1) the maths, (2) how to know what to do if we recognise a similar problem in the future, and (3) how we go about solving problems in general, particularly when we are stuck.

Question 1

(*a*) Given that $\log_4 x = \mathrm{P}$, show that $\log_{16} x = \frac{1}{2}\mathrm{P}$.

(*b*) Solve $\log_3 x + \log_9 x = 12$.

What things did you try? Why?

What makes this problem difficult for you?

What do we know about the log statement $log_4 x = P$?

Choose some value for P, and work out x. Now show that $log_{16} x = \frac{1}{2}P$ for this same value of P and x.

Choose another value for P and do this again. What does this tell you?

How does part (a) help you answer part (b)?

If $log_9(x) = Q$, find the following in terms of Q:

(a) $log_3(x)$

(b) $log_{81}(x)$

(c) $log_{27}(x)$

Create another question like this (parts a and b) and solve it… and a harder one?

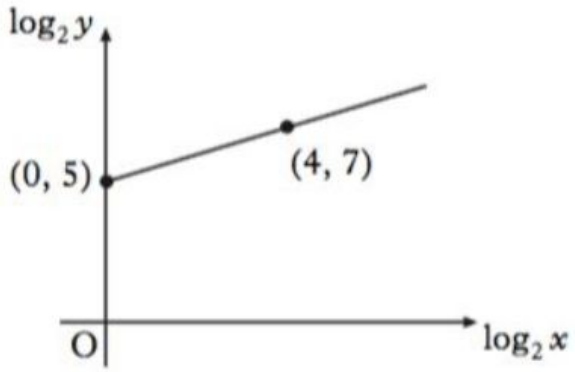
Summarise what you have found out by working on this problem [also: **give it a name**], either about the maths involved, or about solving problems more generally.

Question 2

Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points (0, 5) and (4, 7), as shown in the diagram.

Find the values of k and n.



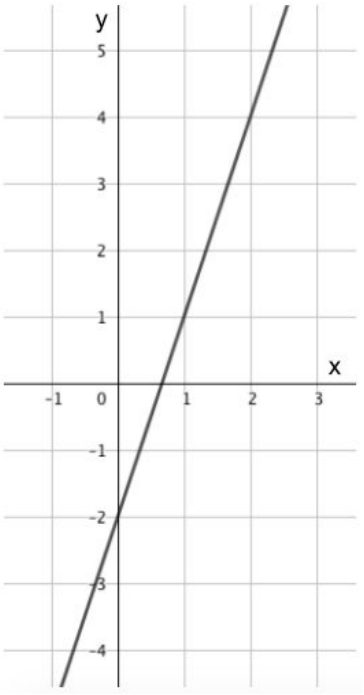
What did you try? Why?

What makes this problem difficult for you?

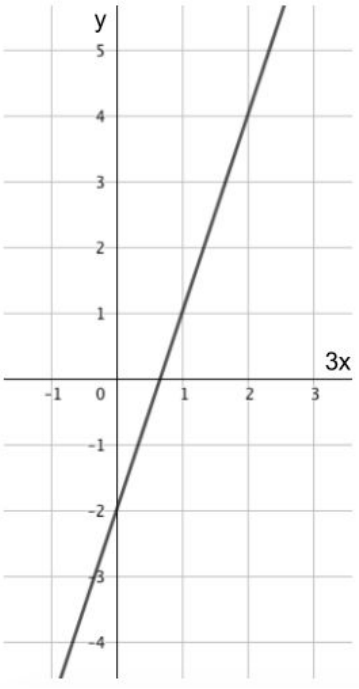
Let's think about what happens when you label the axes with something that is not x and y. Let's look at some similar but easier graphs with unusual axis labels:

Two variables are related by the equation $y = mx + c$. What is the value of m and c for the following graphs:

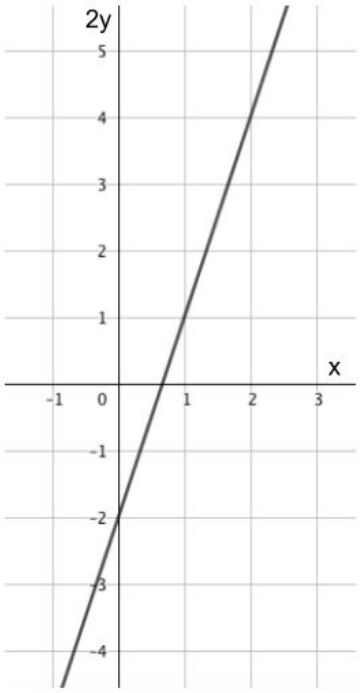
(a)



(b)



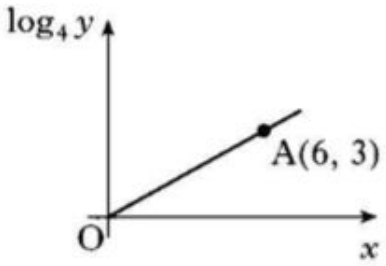
(c)



Can you use the way you worked out the equation of these lines to solve the original problem?

Here is another exam question like this:

Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a.



Create another exam question like this and solve it.

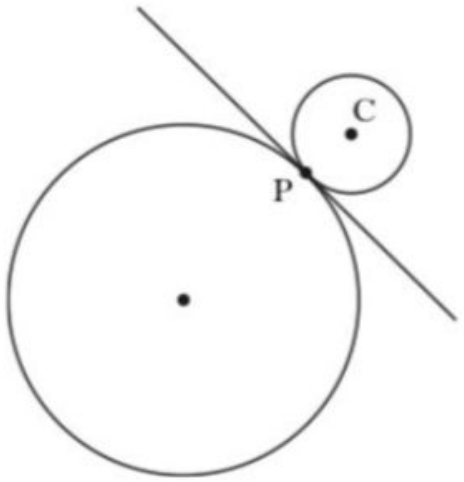
Summarise what you have found out by working on this problem [and give it a name], either about the maths involved, or about solving problems more generally.

Question 3

(*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P.

(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.



The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

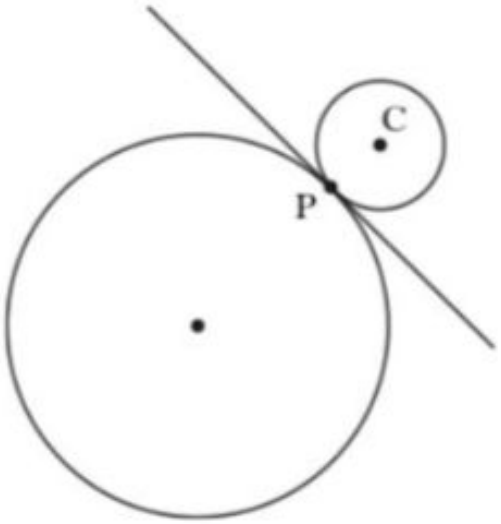
Find the equation of the smaller circle.

What did you try? Why?

What makes this problem difficult for you?

Part (a) is straightforward enough. A good way of solving <u>any</u> pair of simultaneous equations is to make y or x the subject of one equation, and <u>substitute</u> it into the other… try this if you didn't already.

There are easier and harder ways of solving this problem. Often the best way of solving geometric problems is to try to use as little algebra as possible. It is often best to use the diagram to solve the problem.



Start by adding in everything you know onto the diagram (*coordinates of points, radii, etc*…), then say what you see.

Now, are there any additional lines that you can draw on the diagram that might be helpful?

Can you solve it now?

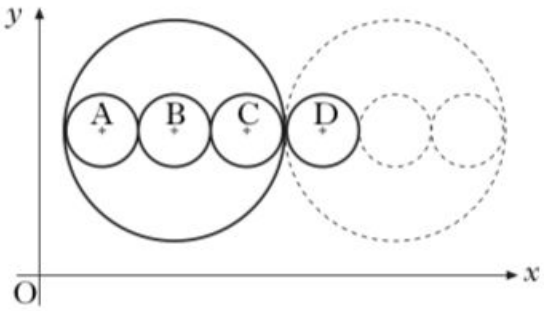
Here is a different but related problem about circles. Can you solve it?

The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



How are these two problems related? What have you learned by working on these problems?

Question 4

Circle C_1 has equation $(x+1)^2+(y-1)^2=121$.
A circle C_2 with equation $x^2+y^2-4x+6y+p=0$ is drawn inside C_1.
The circles have no points of contact.
What is the range of values of p?

What did you try? Why?

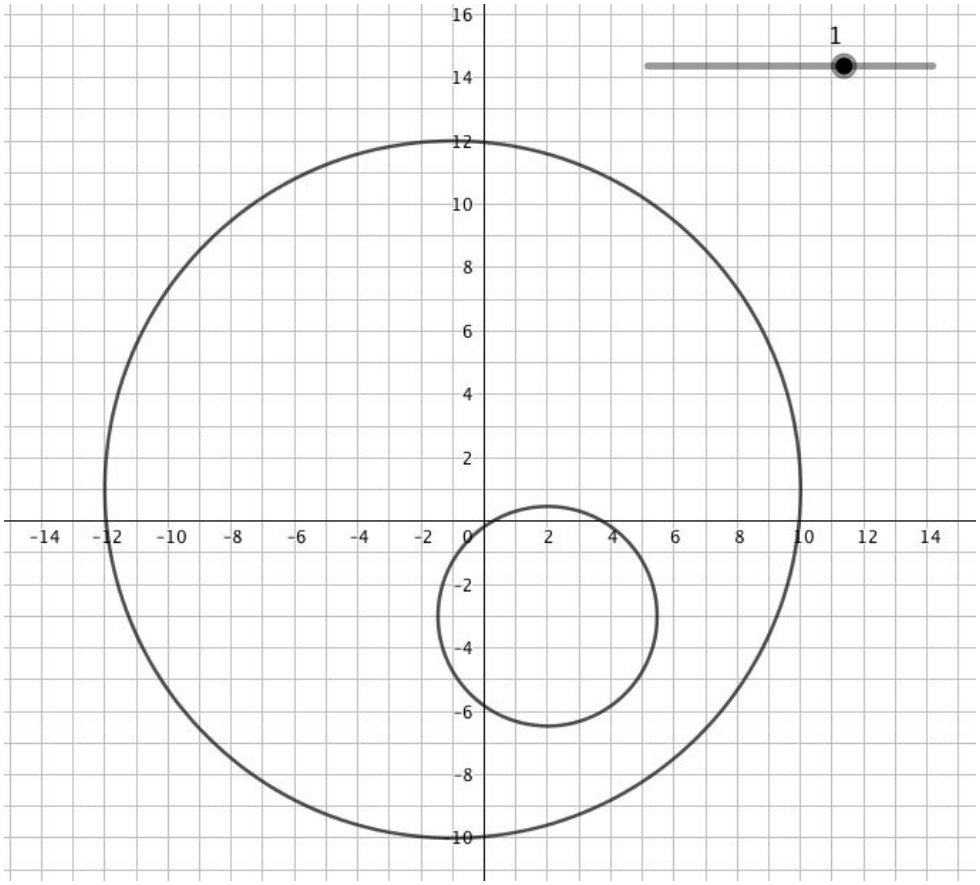
What makes this problem difficult for you?

Do you think algebra is going to be useful in solving this problem?

When solving problems about circles, it is nearly always a good idea to sketch the circles. Make a reasonably accurate sketch of these circles, add on all the information that you know, and add any additional lines onto your sketch that you feel may be useful.

Has this helped? Have you any ideas that might be useful, even if you can't see how to solve the problem directly? Could you use the ideas you learned in **Question 3**?

If you are stuck, draw the two circles on geogebra, and use the slider to change the value of p.



Mark on any information you know on this diagram, any additional lines, etc. Can you see a way of solving this problem now?

This is an unusual question. What have you learned by doing it? Summarise what you have found out by working on all of these circle problems.